Invitation to TDA – Proposed papers

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Presentations should take 30 minutes including time for questions. In case of any questions or if you have problems accessing some paper or if you would like to present some paper that is not on this list, do not hesitate to contact us. The separation into bachelor, master and PhD level is to be taken with a grain of salt, your mileage may vary.

1 Bachelor level

1.1 Simplex Tree

Explain in detail the Simplex Tree datastructure and the costs of operations on it. Sections 2 and 3.1 of [BM14].

1.2 Mapper Algorithm

Explain in detail the Mapper algorithm in [SMC07]. Provide an example of its applications using, for example, the implementation in `kepler-mapper.scikit-tda.org` .

1.3 Persistence Images

Discuss the construction of [Ada+17]. Present of stability results (also look at section 7.1 of [ST20]). Discuss experimental results.

1.4 Persistence Landscapes

Explain the constructions of [BD17], also look at section 7.2 of [ST20] for a discussion of stability.

1.5 Computing Bottleneck Distance

Show how geometry speeds up the Hopcroft-Karp algorithm [EIK01]. Explain how the bottleneck distance computation can be reduced to that problem [KMN17].

1.6 Wasserstein stability I

Prove the cellular Wasserstein stability theorem (=Theorem 4.7) [ST20].

1.7 Approximating Wasserstein

Explain the construction of [CW21] and the approximation guarantees.

2 Master level

2.1 Simplex Tree II

Explain the Critical Simplex Diagram datastructure and how it compares against Simplex Tree. Sections 2, 3 of [JK17] .

2.2 Wasserstein stability II

Prove point cloud Wasserstein stability, section 6 of [ST20].

2.3 Isometry theorem

Explain how morphisms of persistence modules induce matchings of persistence diagrams. Sketch the proof of the main theorem (=Theorem 3.5) in [BL15].

2.4 Statistics with Persistent Homology

Prove the main theorem (=Theorem 5.1) in [Blu+]. Briefly discuss applications.

2.5 ToMATo Clustering

Explain in detail the ToMATo algorithm, part I of [Cha+13]. Either present some of the theoretical results in part II of [Cha+13] or present some experiments using, for example, the implementation in `https://gudhi.inria.fr/python/latest/clustering.html`.

2.6 DTM-Filtration

Construction of the distance-to-measure filtrations [Ana+19], sketch the proof of stability for $p = 1$ (=Theorem 15).

2.7 Vineyards

Explain how to update persistence pairs in linear time, section 3 of [CEM06].

2.8 Persistent homology for images

Explain the T-construction vs. V-construction of cubical complexes from images, the relation of their persistent homology = Theorem 2.1 in [Gar+20].

2.9 Minimal Triangulations

Section 1 of [Lut05] and the references therein.

3 PhD level

3.1 Minimal Persistent Cycles

Explain Algorithm 3.1 of [DHM19].

3.2 Wasserstein stability III

Present the algebraic Wasserstein distance of [ST20]. Prove Theorem 8.26 therein.

3.3 Nerve Theorems

Prove a version of the nerve theorem, e.g. pick one from [Bau+22]. This topic supports possibly more than one presentation.

3.4 Matroid Persistence Algorithm

Explain the persistence algorithm of [HG17] and how to find cycle representatives.

3.5 Computing the Interleaving Distance Is NP-hard

Explain multi-parameter persistence, presentations and the interleaving distance in this setting. Explain the relation to matrix invertibility and NP-hardness.

3.6 Discrete Morse Theory for Geometric Complexes

Show that Alpha and Čech complexes are homotopy equivalent as in Theorem 5.10 of [BE16].

3.7 ZigZag persistence

Motivation: Deletion of simplices. Interval decomposition: Theory and Algorithm [Cd10].

References

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