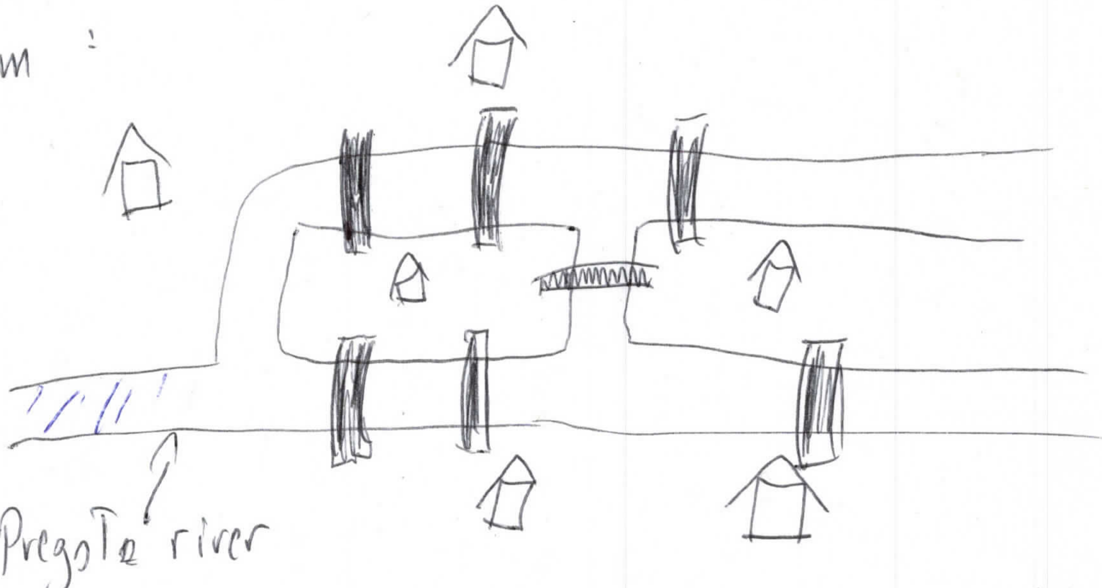
Pawel's Lectures on Applied & Computational Topology

Section 1, the Begining

Topology is a science of proximity, but not metric - it allow to quantify if two points are, in a certain sense, close, without giving the value of distance between them.

Let us consider THE MOST classical example where this philosophy is used; the Königsberg's bridge problem:



Can one start a walk at any point in Königsberg, cross each bridge exactly once & come back to the same spot?

This problem was solved by Leonhart Euler in 1736 & open the door to graph theory, topology, algebraic topology & computational topology.

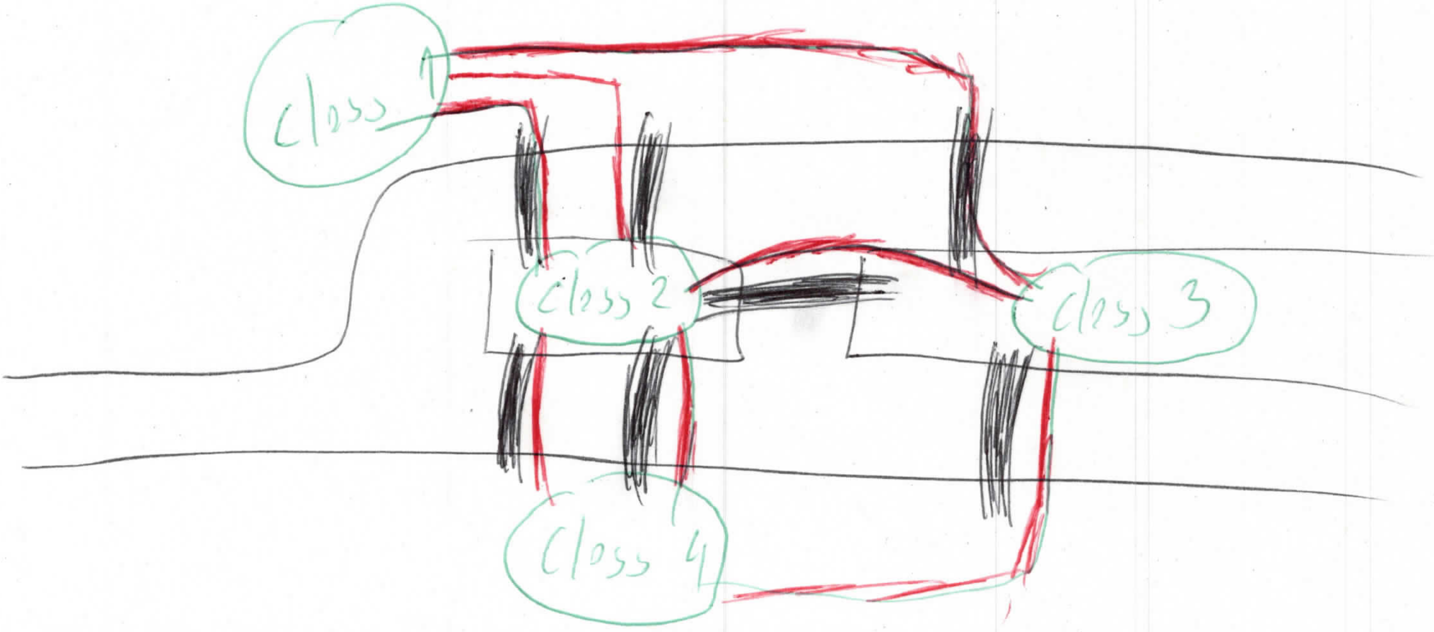
In this case the notion of proximity of two points in the city boils down to the condition that one can be reached from the other without crossing any bridge. That gives an equivalence relation $\sim$:

A, B - points in the map of Königsberg

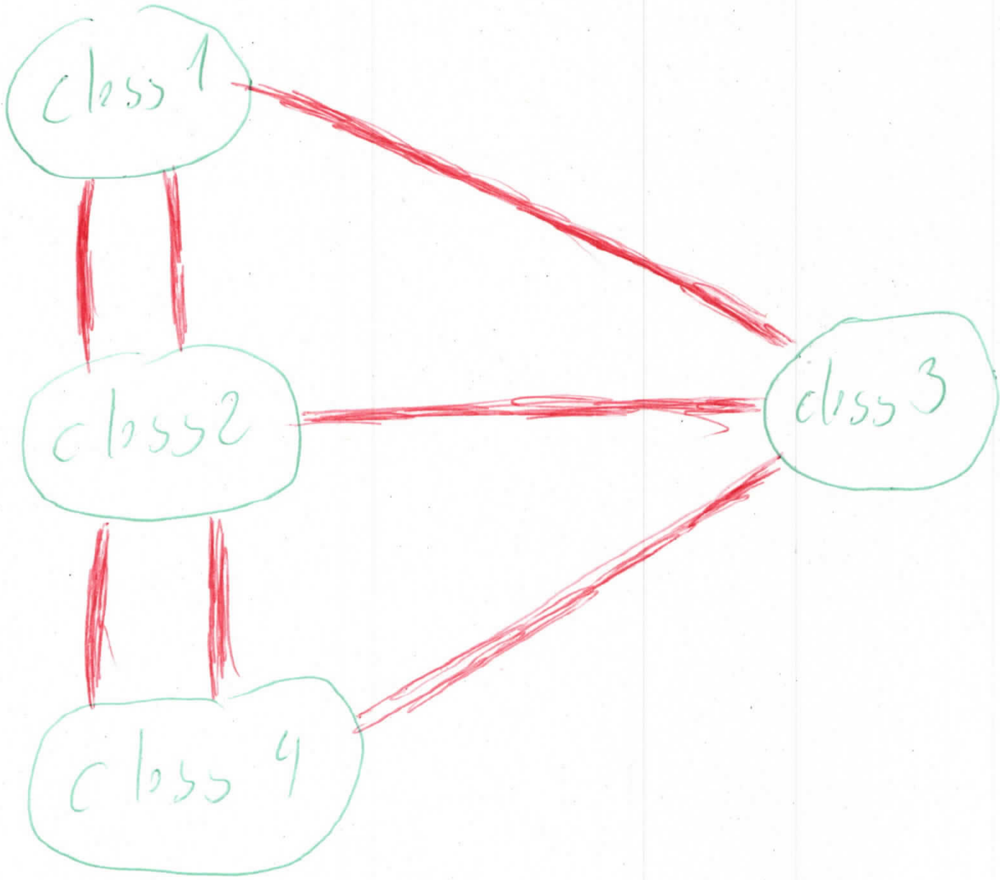
$A \sim B$ iff. there exist a path from A to B that do not cross any bridge

A is close to B

This relation have four classes of abstraction:



That gives an abstract graph:



That gives a graph (or, more presely, a multigraph).
The Königsberg bridge problem can be translated to the following one:

Euler's path.

Is there $\boxed{\text{a path}}$ starting at any vertex in the graph (multigraph) above, crossing each edge <u>only</u> <u>once</u> & coming back to the initial vertex?

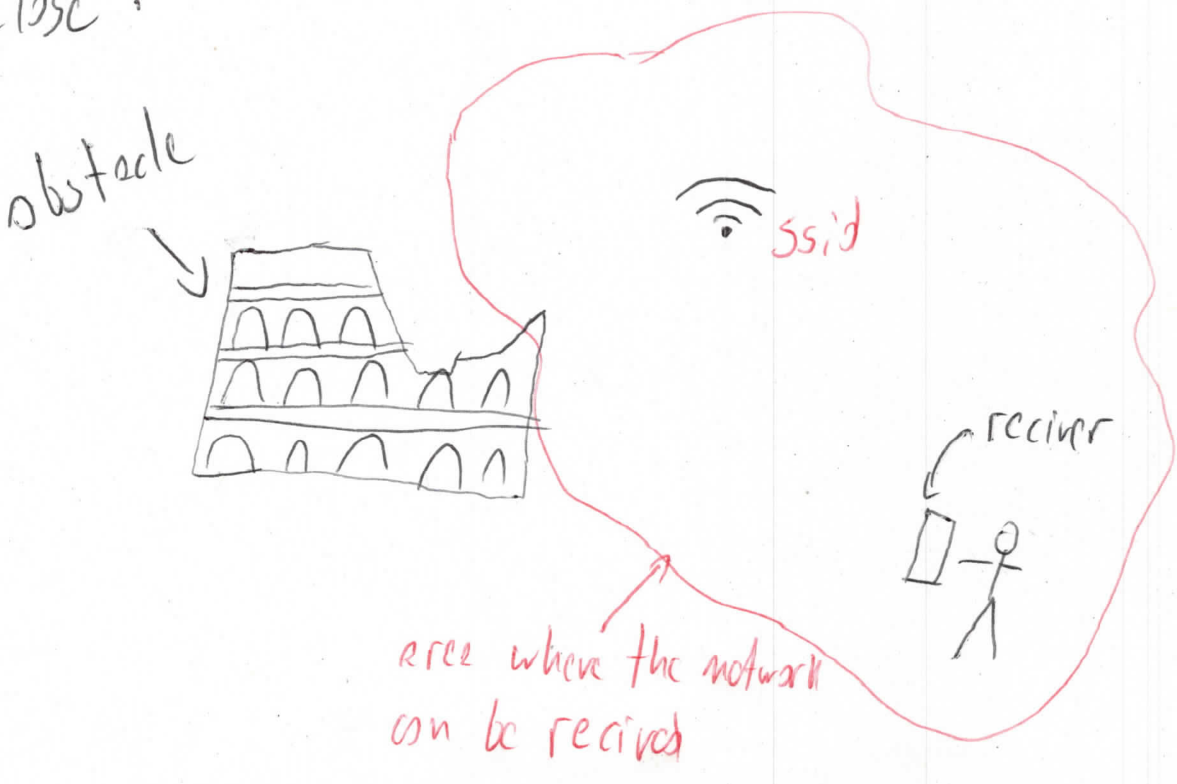
<u>To remember</u>: Notion of proximity was very important ingridient to solve this problem.

Example 2 - remote sensing

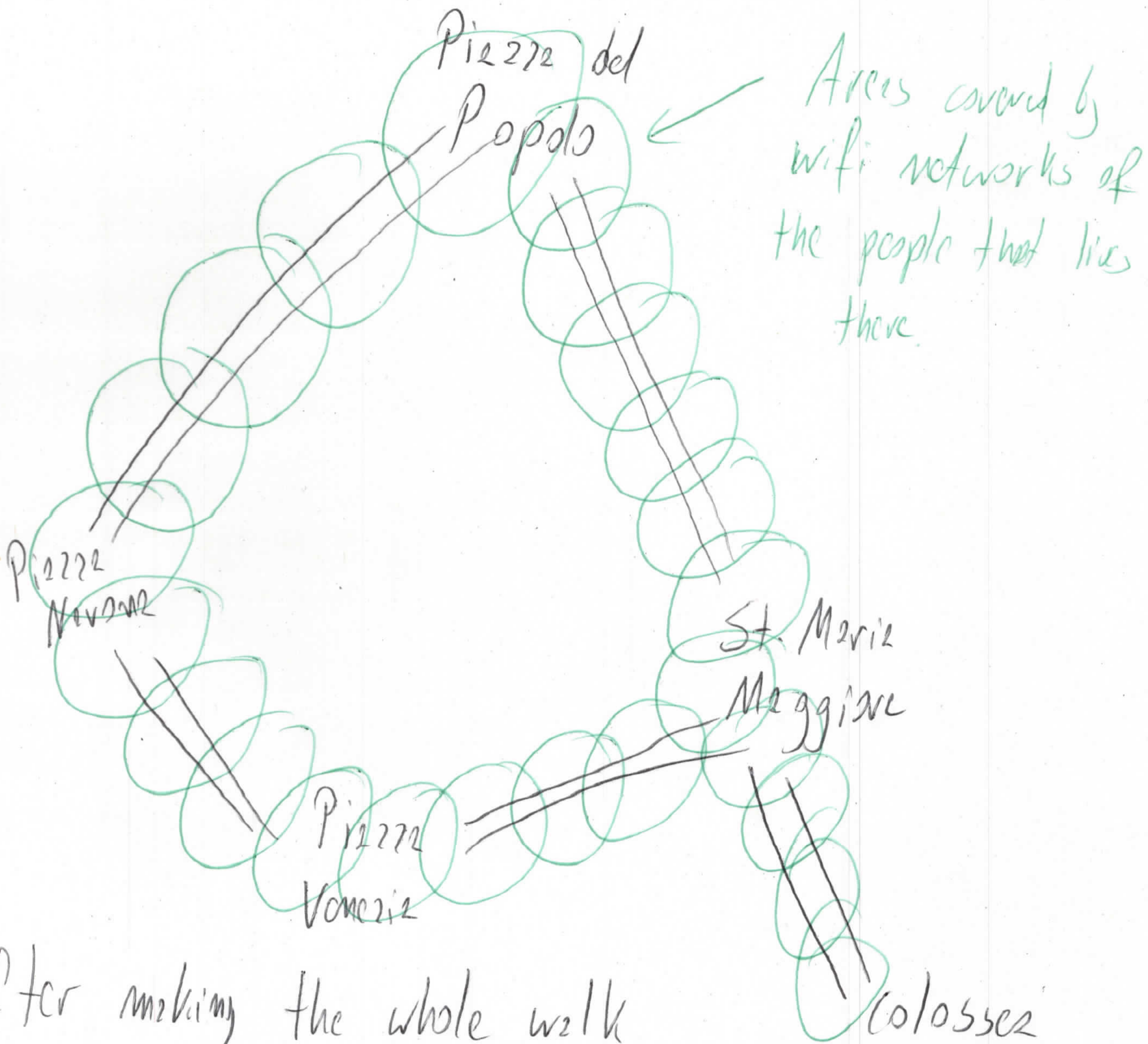
(see the python code at page 15 of my tutorial)

Each of us have a device capable of detecting wifi hotspots nearby. They are all identified by the unique SSID (Service Set Identifier).

When our laptop / phone detect a network of some SSID it means that we are in certain proximity to the corresponding router. But we do not know the distance → it heavilly depends of the strength of the router, obstacles, etc. We only know that we are close:



Imagine now that you walk in an urban area where at every point at least one wifi network is available



After making the whole walk (or shorter...) we have a collection of points in very high dimensional space (= number of registered SSIDs).
Information about router's proximity allow us to recover the overall shape of our walk

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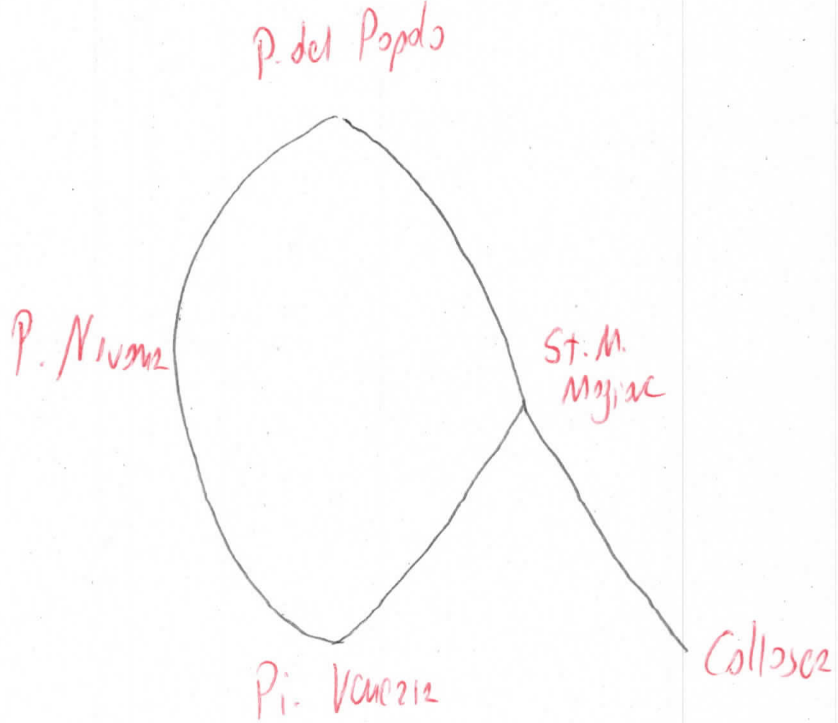

In this context Persistent homology should show one long interval in dimension 1 (corresponding to the cycle).

Mapper should recover a path as the one above (mapper or any other reasonable dimension reduction technique)

When you have time & can take a walk with your laptop try to make similar experiment yourself.

Remember → proximity is defined by region in which a given wifi can be heard

→ Those regions overlap ⇒ global information

Local to global principle in topology.